A1 cont'd.

[of a Minkowski space E] with unit bivector $E = e_\wedge e_*$; and

associating a plurality of general homogeneous operators with each data construct to generate a model of the object.

sub A2

3. The method of claim 1 further comprising:

measuring a scalar distance $\mathbf{d_{ab}}$ between two component points **a** and **b** encoded as [general] homogeneous points a and b by $\mathbf{d_{ab}}^2 = (a-b)^2 = -2a\cdot b$.

4. The method of claim 1 wherein a line through component points **a** and **b** encoded as [general] homogeneous points a and b is modeled by $e_\wedge a_\wedge b$, and a length $\mathbf{l_{ab}}$ of a line segment connecting component points **a** and **b** is generated by:

$$(\mathbf{l_{ab}})^2 = (e_\wedge a_\wedge b)^2 = (a-b)^2.$$

5. The method of claim 1 wherein a plane through component points **a**, **b**, and **c** encoded as [general] homogeneous points a, b, and c is modeled by $e_\wedge a_\wedge b_\wedge c$, and an area $\mathrm{A_{abc}}$ [defined by component points **a**, **b**, and **c**] is generated by $(\mathrm{A_{abc}})^2 = \frac{1}{4}(e_\wedge a_\wedge b_\wedge c)^2$.

6. The method of claim 1 wherein a sphere **s** with radius **r** centered at a component point **c** encoded as a [general homogenous radius r and center] homogeneous point c is [generated by] encoded as a vector $\mathbf{s} = c + \frac{1}{2}r^2 e$.

7. The method of claim 1 wherein a sphere **s** determined by four component points **a, b, c, d** encoded as [general] homogeneous points a,b,c,d is generated by [$\mathbf{s} = IE(a_\wedge b_\wedge c_\wedge d)$, where I is a largest k-blade] $s = (a_\wedge b_\wedge c)(a_\wedge b_\wedge c_\wedge e)^{-1}$.

8. The method of claim [7 wherein one of the general homogeneous] 1 wherein a plane through component points $a,b,$ and $c[,d]$ encoded as homogeneous *points a, b, and c* is [equal to the point e so that **s** defines a plane through the point e] encoded as a vector $p = I(a\wedge b\wedge c\wedge e)|a\wedge b\wedge c\wedge e|^{-1}$, where I is a unit pseudoscalar.

9. The method of claim [5] 8 wherein a distance[s] between a component homogeneous point **a** and a component plane **p** is generated by an inner product[s] $a\bullet p$[of an encoded point a and an encoded plane p].

AU cont.

10. The method of claim 6 wherein a distance[s] between a component homogeneous point **a** and a component sphere **s** is [an] generated by an inner product $a\bullet s$[of and encoded point a and the encoded sphere p].

11. The method of claim 6 wherein a distance between two component spheres [$\mathbf{s_1}$ and $\mathbf{s_2}$ encoded as spheres] $s_1 = c_1 + \frac{1}{2}r_1^2 e$ and $s_2 = c_2 + \frac{1}{2}r_2^2 e$ is generated by $\{[s_1\bullet s_2 = c_1\bullet c_2 + \frac{1}{2}(r_1^2 + r_2^2) = -\frac{1}{2}[(c_1 - c_2)^2 - (r_1^2 + r_2^2)]\}$ $\underline{s_1\bullet s_2 = c_1\bullet c_2 + \frac{1}{2}(r_1^2 + r_2^2)}$ $\underline{= \frac{1}{2}[(r_1^2 + r_2^2) - (c_1 - c_2)^2]}$.

12. The method of claim 1 wherein the object is a rigid body, and a motion of the rigid body is determined by a time dependent displacement versor $D=D(t)$ satisfying a differential equation $\dot{D} = \frac{1}{2}VD$, with "screw velocity" V given by $V = -I\omega + e\mathbf{v}$, where ω is a velocity and $\mathbf{v}$ is a rotational translational velocity of the rigid body.

13. The method of claim 12 wherein *dynamics* of the rigid body are determined by a differential equation $\dot{P} = W$, where $P = -I\mathbf{L} + e_*\mathbf{p}$, and $W = -I\mathbf{T} + e_*\mathbf{F}$, where **L**

is an angular momentum and **p** is a translational momentum of the rigid body, while **T** is [the] a net torque and **F** is a net force on the rigid body.

A2 Cont'd

14. The method[s] of claim 12 wherein the [rigid body includes] object is composed of n linked rigid components, and a motion of the [rigid body] object is modeled by n time dependent displacement versors $D_1, D_2, \ldots, D_n$, [and] with a motion of a k^{th} linked rigid component [is] determined by a versor product $D_1D_2 \ldots D_k$.

15. The method of claim 1 wherein the object[s] is a robot composed of a plurality of [links] rigid bodies connected at joints.

Amended Claims

1. A method for modeling an object composed of one or more components, comprising:

inputting data for each component of the object, the data including coordinates expressed in Euclidean space for a plurality of points **x** of each component;

encoding each point **x** as a null vector x in a homogeneous space by $x = (\mathbf{x} + \frac{1}{2}\mathbf{x}^2 e + e_*)E = \mathbf{x}E - \frac{1}{2}\mathbf{x}^2 e + e_*$, where e and e_* are null vectors with unit bivector $E = e \wedge e$; and

associating a plurality of general homogeneous operators with each data construct to generate a model of the object.

3. The method of claim 1 further comprising:

measuring a scalar distance $\mathbf{d}_{ab}$ between two component points **a** and **b**

encoded as homogeneous points a and b by $\mathbf{d_{ab}}^2 = (a - b)^2 = -2a \bullet b$.

4. The method of claim 1 wherein a line through component points **a** and **b** encoded as homogeneous points a and b is modeled by $e \wedge a \wedge b$, and a length $\mathbf{l_{ab}}$ of a line segment connecting component points **a** and **b** is generated by:

$$(\mathbf{l_{ab}})^2 = (e \wedge a \wedge b)^2 = (a - b)^2.$$

5. The method of claim 1 wherein a plane through component points **a**, **b**, and **c** encoded as homogeneous points a, b, and c is modeled by $e \wedge a \wedge b \wedge c$, and an area $\mathrm{A}_{\mathbf{abc}}$ is generated by $(\mathrm{A}_{\mathbf{abc}})^2 = \frac{1}{4}(e \wedge a \wedge b \wedge c)^2$.

6. The method of claim 1 wherein a sphere **s** with radius **r** centered at a component point **c** encoded as a homogeneous point c is encoded as a vector $\mathbf{s} = c + \frac{1}{2}r^2 e$.

7. The method of claim 1 wherein a sphere **s** determined by four component points **a, b, c, d** encoded as homogeneous points a,b,c,d is generated by $s = (a \wedge b \wedge c)(a \wedge b \wedge c \wedge e)^{-1}$.

8. The method of claim 1 wherein a plane through component points a,b, and c encoded as homogeneous points a, b, and c is encoded as a vector $p = I(a \wedge b \wedge c \wedge e)|a \wedge b \wedge c \wedge e|^{-1}$, where I is a unit pseudoscalar.

9. The method of claim 8 wherein a distance between a component homogeneous point **a** and a component plane **p** is generated by an inner product $a \bullet p$.

10. The method of claim 6 wherein a distance between a component homogeneous point **a** and a component sphere **s** is generated by an inner product $a \bullet s$.

11. The method of claim 6 wherein a distance between two component spheres $s_1 = c_1 + \frac{1}{2} r_1^2 e$ and $s_2 = c_2 + \frac{1}{2} r_2^2 e$ is generated by $s_1 \bullet s_2 = c_1 \bullet c_2 + \frac{1}{2}(r_1^2 + r_2^2) = \frac{1}{2}[(r_1^2 + r_2^2) - (c_1 - c_2)^2]$.

12. The method of claim 1 wherein the object is a rigid body, and a motion of the rigid body is determined by a time dependent displacement versor $D=D(t)$ satisfying a differential equation $\dot{D} = \frac{1}{2} VD$, with "screw velocity" V given by $V = -I\omega + e\mathbf{v}$, where ω is a velocity and $\mathbf{v}$ is a rotational translational velocity of the rigid body.

13. The method of claim 12 wherein *dynamics* of the rigid body are determined by a differential equation $\dot{P} = W$, where $P = -I\mathbf{L} + e_*\mathbf{p}$, and $W = -I\mathbf{T} + e_*\mathbf{F}$, where $\mathbf{L}$ is an angular momentum and $\mathbf{p}$ is a translational momentum of the rigid body, while $\mathbf{T}$ is a net torque and $\mathbf{F}$ is a net force on the rigid body.

14. The method of claim 12 wherein the object is composed of n linked rigid components, and a motion of the object is modeled by n time dependent displacement versors $D_1, D_2, \ldots, D_n$, with a motion of a k^{th} linked rigid component determined by a versor product $D_1 D_2 \ldots D_k$.

15. The method of claim 1 wherein the object is a robot composed of a plurality of rigid bodies connected at joints.